

# A hybrid Genetic Algorithm in PBRDF modeling

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**Abstract**—The polarized light scattered by the surface of a material contains information that can be used to describe the properties of the surface. Polarized Bidirectional Reflectance Distribution Function (PBRDF) is one of the most important factors used to represent the property of the surface. Because there is complex nonlinear relationship between the experimental results and model parameters, genetic algorithm is used to retrieve the model parameters. One drawback of the traditional genetic algorithm is that the convergence speed is slow and easy to fall into the local minimization. On the base of the traditional genetic algorithm to retrieve the parameters, simulated annealing (SA) algorithm is used to optimize the modeling of the PBRDF. The model for PBRDF and the designation of the hybrid algorithm is given in detail. For one typical painted surface, both the experiment results and the model calculation results are given. The calculation results of the model are demonstrated consistent well with the experimental results. The error convergence curve shows that, the hybrid genetic algorithm can avoid falling into the local minimization, and shorten the running time for the target function. Therefore, it is applicable used as a reference for target feature extraction and recognition in the future.

**Keywords**- Genetic algorithm, light scattering, Simulated annealing, Polarized Bidirectional Reflectance Distribution Function (PBRDF)

## 1 INTRODUCTION

In the target stealthy and camouflage, the surfaces of the targets are often painted with various paintings. These paintings have different color and refractive index which make the scattering properties of the surfaces more and more complicated.

The conventional method used to describe the reflected component of the radiance from a surface is the Bidirectional Reflectance Distribution Function (BRDF) defined by Nicodemus<sup>[1]</sup> and the BRDF is defined as:

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{dL_r(\theta_i, \phi_i, \theta_r, \phi_r)}{dH_i(\theta_i, \phi_i)} (sr^{-1}) \quad (1)$$

Measurement of BRDF<sup>[2-5]</sup> and the analysis in terms of BRDF are commonplace. Unfortunately, the BRDF description does not properly account for the polarization characteristics of the reflected radiation. A key factor to describe painted objects is the polarized BRDF. The PBRDF contains more information than that of the unpolarized BRDF and it can be used for target surveillance and recognition system<sup>[6,7]</sup>.

In our previous work, a PBRDF model based on microfacet theory<sup>[7,8]</sup> was employed in the application<sup>[9,10,11]</sup>. The polarized model is determined by the roughness parameters, optical constant parameters, the incident and viewing angles (including the zenith and azimuth angles) together. For there is a complex non-linear relationship between the model parameters and target output, it is hard for the traditional linear fitting method to obtain the parameters.

Genetic algorithm (GA) is a probabilistic search method that employing a search technique based on the idea from natural genetics and evolutionary principles. They were conceived by Holland<sup>[12]</sup>, they have emerged as general purpose, robust optimization techniques<sup>[13,14,15,16]</sup> in the practical application. But for some complicated functions, the convergence speed of GA is slow and it is not guaranteed to find the global functional optima. Simulated annealing (SA) is another important algorithm which is powerful in optimizing and solving high-order problems. SA is a kind of single point based search strategy that is an iterative improvement scheme with hill-climbing ability, which allows it to reject inferior local solutions and find more globally near-optimal solutions. Similarly, it does not guarantee to find the global functional optima as well. But if the function optimization problem has many good near-optimal solutions, SA should find one near-optimal solution.

In this paper, a hybrid algorithm based on GA and SA is proposed to deal with the optimized solution to the PBRDF model which can make the convergence speed and accuracy more satisfactory. The hybrid algorithm is based on basic GA, and in the fitness calculation procedure, the SA algorithm is used to improve the convergence efficiency, and therefore the hybrid algorithm can guarantee the diversity of the solution and can avoid falling into the local minimum.

## 2. THEORY FOR POLARIZED BRDF

The function used to describe the directional dependence of the reflected energy from the surface is the BRDF first proposed by Nicodemus<sup>[1]</sup> in 1977. The geometry of the BRDF definition is shown in figure 1. The BRDF is defined as the ratio of the radiance ( $W/(m^2 \cdot sr)$ ) in the outgoing direction  $(\theta_r, \phi_r)$ , to the irradiance ( $W/m^2$ ) that reaches the surface from incoming direction  $(\theta_i, \phi_i)$ . It depends on the incoming zenith angle and azimuth angle, outgoing zenith angle and azimuth angle, and the wavelength. The definition formulation of BRDF is:

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{dL_r(\theta_i, \phi_i, \theta_r, \phi_r)}{dE_i(\theta_i, \phi_i)} (sr^{-1}) \quad (1)$$

$\theta$  and  $\phi$  represent the zenith and azimuth angle respectively, and  $Z$  represents the surface normal. The subscript  $r$  and  $i$  represent the reflection light and the incident light respectively.

Based on the geometry optics, the micro-facet model assumes that the surface consists of small, randomly disposed, mirror like facets. Each micro-facet is a specular reflector obeying Snell's law of reflection with reflectivity given by a Fresnel reflectivity<sup>[7,8]</sup>. The geometry of reflection is shown in figure 1.  $N$  is the unit normal to the surface,  $k_i$  is the unit vector in the specific light source,  $k_r$  is the unit vector in the direction of the viewer and  $H$  is a normalized vector in the direction of the angular bisector of  $k_r$  and  $k_i$ . The polarized BRDF is based on electronic theory, the incident electric field is described by the components of the electric field along  $\vec{s}_i$  and  $\vec{p}_i$  directions, where  $\vec{s}_i$  is a unit vector perpendicular to both  $\vec{k}_i$  and  $Z$ . Likewise, the polarization of the electric field in the viewer direction is described by components along the  $\vec{s}_r$  and  $\vec{p}_r$ , and

$$\vec{p}_i = \vec{k}_i \times \vec{s}_i, \quad \vec{p}_r = \vec{k}_r \times \vec{s}_r \quad (2)$$

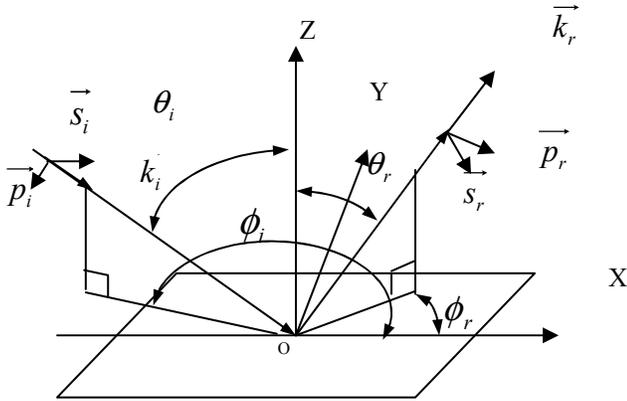


Figure 1. Polarized BRDF coordination

The relationship among  $\theta, \beta, \theta_i, \theta_r, \phi_i, \phi_r$  is:

$$\cos(2\beta) = \cos(\theta_i)\cos(\theta_r) + \sin(\theta_i)\sin(\theta_r)\cos(\phi_r - \phi_i) \quad (3)$$

$$\cos(\alpha) = \frac{\cos(\theta_i) + \cos(\theta_r)}{2\cos(\beta)} \quad (4)$$

The scattering matrix (Jones matrix) is defined as the relationship between the incident and scattered fields<sup>[9]</sup>:

$$\begin{pmatrix} E_p^{scat} \\ E_s^{scat} \end{pmatrix} = \begin{pmatrix} S_{pp} & S_{sp} \\ S_{ps} & S_{ss} \end{pmatrix} \begin{pmatrix} E_p^{inc} \\ E_s^{inc} \end{pmatrix} \quad (5)$$

There is a relationship between Jones matrix and Mueller matrix<sup>[6]</sup>, and Mueller matrix contains the  $n$  and  $k$  coefficients which are determined by the Fresnel equation. So the PBRDF model can be written as the following<sup>[9]</sup>:

$$f_{j,k}(\theta_i, \theta_r, \phi_r - \phi_i) = \frac{1}{2\pi} \frac{1}{4\sigma^2} \frac{G(\theta_i, \theta_r)}{\cos^4(\theta)} \frac{\exp(-\frac{\tan^2(\theta)}{2\sigma^2})}{\cos(\theta_r)\cos(\theta_i)} M_{j,k}(\theta_i, \theta_r, \phi_r - \phi_i) \quad (6)$$

$j$  and  $k$  range from 0 to 3. Considering the effects of the diffuse scattering from the surface, we add the coefficients of  $k_s$  and  $k_d$  to adjust the model:

$$f_{j,k}(\theta_i, \theta_r, \phi_r - \phi_i) = k_s \frac{1}{2\pi} \frac{1}{4\sigma^2} \frac{G(\theta_i, \theta_r)}{\cos^4(\theta)} \frac{\exp(-\frac{\tan^2(\theta)}{2\sigma^2})}{\cos(\theta_r)\cos(\theta_i)} M_{j,k}(\theta_i, \theta_r, \phi_r - \phi_i) + \frac{k_d}{\cos(\theta_i)} \quad (7)$$

Therefore, once the parameters of  $n, k$  (both  $n$  and  $k$  are contained in the elements of  $M_{j,k}(\theta_i, \theta_r, \phi_r - \phi_i)$ ),  $k_s, k_d$  and  $\sigma$  are determined, the distribution of the PBRDF can be known.

### 3. THE EVALUATION FUNCTION

The criterion for the determination of the model parameters is how to set a suitable evaluation function to get the most optimized solution. The evaluation function is the following:

$$\Delta_{\min} = \frac{\sum_{\theta_i, \theta_r} \sum_{\phi_i, \phi_r} g(\theta_r) (f_{00}(\theta_r, \theta_i) - f_r(\theta_r, \theta_i))^2}{\sum_{\theta_i, \theta_r} \sum_{\phi_i, \phi_r} g(\theta_r) (f_r(\theta_r, \theta_i))^2} \quad (8)$$

Where  $f_{00}(\theta_r, \theta_i)$  is the model calculation data, and  $f_r(\theta_r, \theta_i)$  is the experimental data,  $g(\theta_r)$  is the weight coefficient for different sampling step. Through many times iteration, when  $\Delta_{\min}$  is convergent to sufficiently low, the optimized or approximately optimized parameters can be determined.

### 4. THE HYBRID GENETIC ALGORITHM FOR MODEL PARAMETER RETRIEVING

Evolution by natural selection is one of the most compelling themes of modern science, Genetic Algorithm<sup>[13]</sup> is a form of evolution, which can search the optimized solution globally. In the application, the optical constants  $n$  and  $k$  for rough surfaces which have complicated paintings are difficult to measure. The statistical method is more convenient to obtain the model parameters for the experimental data. In general,

least square fitting and polynomial fitting methods are often adopted to get the statistical parameters [17,18]. However, it is difficult for the traditional methods to offer appropriate parameters for the model because of the complicated nonlinear relationship between target solution and input data. A hybrid GA is thus employed to get the parameters for the PBRDF model in the parameters retrieving. The proposed statistical method is more straightforward and much simpler compared with traditional methods in which the ellipsometer was used to get the optical constant and the roughness parameters.

Figure 2 shows the flow chart of the GA. The first step of the algorithm is the parameters initialization, and an initialized population is produced which concluded the initialized temperature and ending temperature. For one painted sample, the population is 100, the initial temperature is 2000(which is an arbitrary unit for temperature), and the ending temperature is 0.002. Function (8) is used as evaluation criteria. In the Genetic operation, and crossover probability is setting as 0.5, and the mutation probability is setting as 0.05. The proportion method is used as selection algorithm, and the binary method is used as coding method. The new population which was produced by GA is used as the initial population of the SA. The acceptance criterion for hybrid algorithm is Metropolis method [19]. Suppose the variance for optimization is  $M$ , the initialization state is  $M^0$ , the new state (the nearby state) is  $M^N$ , and the corresponding target function are  $E^0$  and  $E^N$  respectively. The difference between  $E^0$  and  $E^N$  is  $\Delta E$  which can be written as  $\Delta E = E^N - E^0$ . If  $\Delta E < 0$ , the target function is descended, which means the new  $M^N$  is acceptable, and the solution can be accepted. Conversely, if  $\Delta E \geq 0$ , the target function is ascended, the acceptance probability is  $P = e^{-\Delta C/T}$ , in which T is the annealing temperature. In the following, a random number R can be produced between 0 and 1 and was taken comparison with P. If  $P \geq R$ , the new solution  $M^N$  can be accepted, otherwise, the solution will be discarded. The temperature descends method for annealing algorithm is exponential mode, supposing the temperature at time t is  $T(t)$ ,  $T(t) = T_0 / \lg(1+t)$ , in which  $T_0$  is the initial temperature. For the annealing operation is included in the GA, two termination conditions for searching the optimized solution are setting as the followings: 1) the  $\Delta_{\min}$  is small enough and 2) the annealing temperature is low enough. If either of the condition is satisfied, the optimized program will be finished.

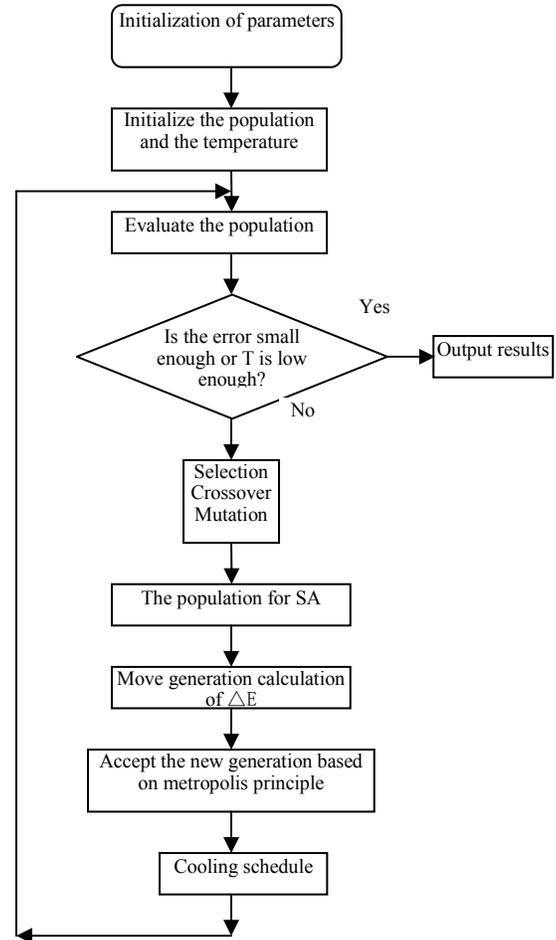


Figure 2. Flow chart of the hybrid genetic algorithm

### 5. THE HYBRID MODEL CALCULATION RESULTS AND COMPARISON WITH EXPERIMENTAL DATA

For one painted sample, we took 10 groups data got from the BRDF measurement instrument [2] in Anhui Institute Optical and Fine Mechanics, CAS to retrieve the PBRDF parameters. The measurement conditions are the receiving azimuth angle  $\phi_r = 0^\circ$ , the receiving zenith angle  $\theta_i = 45^\circ$  at  $0^\circ, 5^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 45^\circ, 50^\circ, 55^\circ, 60^\circ$  respectively, and the laser wavelength is  $1.06 \mu m$ . The weight factor  $g(\theta_r, \theta_i)$  is increased when the angle range in  $\pm 5^\circ$  is at the near specular direction where the acquisition intensity is much higher.

Table 1 shows the comparison of the results between basic Genetic Algorithm (GA) and hybrid genetic algorithm (SA-GA), both of which utilize the same sample and the same target function. Seen from table 1, not only the iteration number for SA-GA is smaller than GA, but also the ultimate convergence error for SA-GA is smaller than GA. Figure 3 shows the error convergence plot comparison between GA and SA-GA. The convergence gradient for SA-GA is much bigger than that of SA, which can effectively avoid falling into the local minimum (The error for SA is 0.023562 which is a local minimum).

Table 1 Comparison of the PBRDF parameters between GA and SA-GA

Parameter	GA	SA-GA
Loop number	100	78
error	0.023562	0.023518
ks	0.949726	0.950000
kd	0.092414	0.092493
n	5.345064	5.372434
k	5.345064	5.372434
$\sigma$	0.123451	0.123724

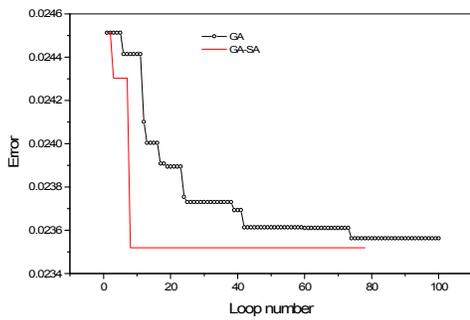


Figure 3. Error comparison between GA and SA-GA algorithm

According to the parameters attained through SA-GA algorithm, the  $f_{00}$  can be calculated using the formula (7), Figure 4 presents the comparison between the experimental data and simulation results. Where figure A、B、C、D is  $\phi_r = 0^\circ, \theta_i$  is  $15^\circ, 25^\circ, 30^\circ, 45^\circ$  respectively, in figure 4, the dot curves represent the experimental data and the line curves represent the simulation results, the simulation results agree well with the experimental data. It's noticed that, the data which the incidence angle  $\theta_i$  is  $15^\circ$  and  $25^\circ$  is not be included in the parameter retrieving, the simulation results can agree well with the experimental data. The comparison indicates, this kind of hybrid genetic algorithm to retrieve the parameters from large-scale data can be extended to other angles which can be used as a method in the polarization BRDF modeling.

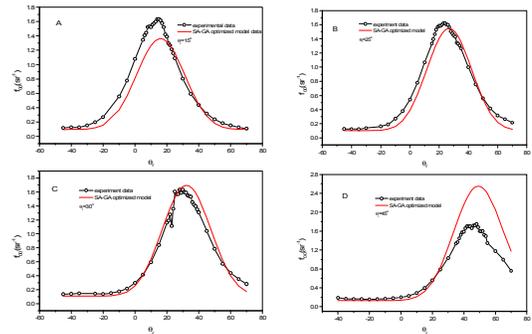


Figure 4. Comparison between experimental data and simulation results with different incident angle

## 6. CONCLUSIONS AND FUTURE WORK

Based on the basic Genetic algorithm theory, a hybrid algorithm which combining the merit of both Genetic algorithm and simulated annealing algorithm was used to retrieve the parameters contained in the model from large-scale experimental data. The simulation results agree well with the experimental data. It shows that this statistical method to retrieve the parameters in the modeling is convenient for some complex painted surface whose optical constant is hard to be measured exactly. The experimental data used in the retrieving is the  $f_{00}$  data; more data is needed when more precise model parameters are expected.

## ACKNOWLEDGMENT

This work was supported by State Key Development Program for Basic Research of China (61341020201-1) and the Department of Science and Technology of Shandong Province (2008GG20005005)

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