Oil Slick Observation at Low Incidence Angles in Ku-Band

M. A. Panfilova¹, V. Y. Karaev¹, and Jie Guo²

¹Institute of Applied Physics of Russian Academy of Sciences, Nizhny Novgorod, Russia, ²Key Laboratory of Coastal Zone Environmental Processes, CAS; Shandong Provincial Key Laboratory of Coastal Zone Environmental Processes, Yantai Institute of Coastal Zone Research, Chinese Academy of Sciences, Yantai, China

Abstract On the 20 April 2010 the oil platform Deep Water Horizon in the Gulf of Mexico suffered an explosion during the final phases of drilling an exploratory well. As a result, an oil film covered the sea surface area of several thousand square kilometers. In the present paper the data of the Ku-band Precipitation Radar, which operates at low incidence angles, were used to explore the oil spill event. The two-scale model of the scattering surface was used to describe radar backscatter from the sea surface. The algorithm for retrieval of normalized radar cross section at nadir and the total slope variance of large-scale waves compared to the wavelength of electromagnetic wave (22 mm) was developed for the Precipitation Radar swath. It is shown that measurements at low incidence angles can be used for oil spill detection. This is the first time that the dependence of mean square slope of large-scale waves on wind speed has been obtained for oil slicks from Ku-band data, and compared to mean square slope obtained by Cox and Munk from optical data.

1. Introduction

Usually oil slicks are not large and high spatial resolution is required for their observation. Synthetic Aperture Radar (SAR) has been widely used for slick detection, e.g., Kudryavtsev et al. (2013), Zhang et al. (2011), Singha et al. (2016). The results of measurements at low incidence angle in the area of oil slick were presented in Johnson and Crosswell (1982). In this paper the data of spaceborne Ku-band Precipitation Radar (PR) (Kummerow et al., 1998) which operates at low incidence angles (6–18.1°) are used in the area of the large oil spill in the Gulf of Mexico from April to July 2010. This radar was part of the instrument package on the Tropical Rainfall Measurement Mission (TRMM) satellite. TRMM’s main mission was to measure spatial distribution of precipitation in tropical zone (intensity of rain was determined from the value of volume scattering measured with resolution 250 m by height). It is also possible to retrieve information on backscatter from the underlying surface from PR data. The normalized radar backscattering cross section (NRCS) measured by PR at different sea states has been the subject of several research efforts (Chu & Chen, 2010; Chu et al., 2012a, 2012b; Freilich & Vanhoff, 2003; Li et al., 2002). For interpretation of microwave radar measurements at low incidence angles the Geometrical Optics (GO) model is widely used (Barrick, 1968; Bass & Fuks, 1972; Valenzuela, 1978). In the framework of the two-scale model the sea surface consists of smoothed large-scale (compared to the microwave sensing wavelength) waves and small-scale ripples riding on them (Bass & Fuks, 1972; Valenzuela, 1978). The surface elevation spectrum \( S(\kappa) \), where \( \kappa \) is a wave number, is divided into the large-scale and small-scale parts by the boundary wave number \( \kappa_B \). For the large-scale surface the following conditions are satisfied

\[
\Lambda \gg \lambda \quad (2\pi/\lambda)R_c \cos \theta \gg 1.
\]

where \( \lambda \) is the length of microwave sensing wave, \( \Lambda \) is the length of large-scale sea wave, \( \theta \) is the incidence angle, and \( R_c \) is the mean curvature radius. In Lementa (1980) and Kanevsky (2009), \( R_c \) is calculated as follows

\[
R_c = \left[ \int_0^{\kappa_B} \kappa^4 S(\kappa) d\kappa \right]^{-1/2}.
\]

According to GO approximation, NRCS at low incidence angles is proportional to the probability distribution of the large-scale surface slopes. If such a distribution is assumed to be Gaussian, it explicitly depends on
the mean square slope of large-scale surface \((mss_{LS})\). The parameter plays an important role in the backscatter of microwaves by the ocean surface, and has been investigated in field experiments, for example in Danilytchev et al. (2009). Observations in microwave range at low incidence angles can only be used to obtain the “radar filtered” \(mss_{LS}\), that is the mean square slope of fraction of waves in the spectrum with \(\kappa < \kappa_0\), where \(\kappa_0\) depends on the incident electromagnetic wavelength.

The effect of small-scale part of the surface is taken into account by introducing the effective reflection coefficient instead of the Fresnel coefficient. In Freilich and Vanhoff (2003), PR data were used in making assessments of \(mss_{LS}\) and the effective nadir reflectivity for different wind speeds, using the GO model and in the framework of the two-scale model of the sea surface. Also the dependence of NRCS on wind speed at low incidence angles was discussed. In Chu et al. (2012a), the correlation between NRCS and wind speed was studied for different incidence angles and at various sea states. In Chu et al. (2012b), the asymmetry and anisotropy of Ku-band NRCS was studied. Also in Chu et al. (2012b) the dependences of coefficients for quasi-gaussian distribution obtained from PR data on wind speed are presented. PR data were used for wind speed retrieval in Li et al. (2002). In Chu and Chen (2010), NRCS at nadir and \(mss_{LS}\) were obtained and applied in the two-parameter algorithm for the retrieval of wind speed. Though NRCS at nadir and \(mss_{LS}\) were defined from PR data in previous works, the spatial distribution of these parameters within the radar swath has not received attention to date. In the present paper we use PR data to obtain the spatial distribution (map) of nadir NRCS and total \(mss_{LS}\) within the PR swath, so that the angle dependence in NRCS data is removed and large-scale processes on the ocean surface can be explored. The algorithm for calculation of nadir NRCS and total \(mss_{LS}\) is thoroughly discussed. Data for the area of oil spill on the Gulf of Mexico were chosen to validate the algorithm developed. Distributions of nadir NRCS and mean square slope are correlated to microwave and optical images of the oil spill where suppression of short waves is anticipated. In section 2, information on the PR is given, in section 3 the GO model for near-nadir backscatter is discussed. In section 4, the method for retrieval of NRCS at nadir and \(mss_{LS}\) along the direction of scanning is presented. Section 5 gives some estimates of the method accuracy using numerical simulation. The empirical algorithm for evaluation of total \(mss_{LS}\) from NRCS at nadir is outlined in section 6. In section 7, the distribution (map) of the NRCS measured at different incidence angles (ranging from 0° to 18.1°) in presence of oil spill and the dependence of contrast on incidence angle are presented. In section 8, examples of NRCS at nadir and total \(mss_{LS}\) distributions within the PR swath in the area of oil spill are considered, and the dependence of \(mss_{LS}\) on wind speed in the slick for Ku-band is presented. The meteorological buoy 42040 (coordinates 29.208°N, 88.226°W) is located 54 km from the Deep Water Horizon platform (coordinates 28.737°N, 88.366°W). The wind speed data from this buoy were used to obtain the dependence of total \(mss_{LS}\) on wind speed in the slick for Ku-band, which is compared to the result of well-known Cox and Munk (1954) experiment for the optical frequency band in slicks.

2. TRMM Ku-Band Precipitation Radar

The TRMM satellite was launched in November 1997, carrying five instruments including the PR, and ceased to provide the data in 2014. Since the main objective of TRMM was to measure rainfall in the tropics, the satellite ground track is confined between 35°S and 35°N. The PR is a Ku-band pulsed radar with horizontal polarization. The PR antenna scans perpendicularly to the flight direction. The scanning angle varies from −17° to +17°, with 49 beam positions separated by 0.71°. The scanning scheme is shown in Figure 1. The scan duration is equal to 0.6 s with the antenna footprint size for the original orbit height being 4.3 km in both the along and cross-track directions. After the orbit boost of TRMM from 350 to 403 km in August 2001 the spatial resolution of the PR became 5.0 km by 5.0 km. The data product used in the present paper is TRMM PR standard product 2A21. These data include normalized radar cross sections, local incidence angles and the antenna footprints coordinates. The local incidence angle was calculated taking into account the shape of Earth. Maximum local incidence angle is 18.1°. Also, the data contain a rain
flag for each incidence angle bin or pixel. Data over land or with rain over the ocean are excluded from further processing.

The measurements are conducted at the two close frequencies 13.796 GHz and 13.802 GHz. For each cell (beam position) 32 pulsed on each frequency are emitted. Pulse repetition frequency is 2,776 Hz and each pulse is 1.6 mcs long. After that 64 independent pulses are averaged in order to reduce statistical fluctuations of the signal. These fluctuations (that we will further refer to as “noise”) are caused by coherent summation of complex signals with different phases reflected from the scatterers (the regions of wave profile oriented perpendicular to the incident radiation).

The values of NRCS measured by PR are provided as discretized values, with intervals of 0.35 dB. For example the histogram of NRCS for one of incidence angles (θ=7.5°) is presented in Figure 2.

3. Model for Backscatter at Low Incidence Angles

The electromagnetic wave is incident to the sea surface at the angle θ, and the wave vector of incident wave is \( \vec{q} \). The axis X lies in the plane of incidence, and Y is perpendicular to X; X' and Y' are directions along wind and across wind respectively. The axes X, Y, X' and Y' lie in the horizontal plane. Let us define the X, Y, X', and Y' components of unit vector along the scanning direction \( \tau_x \), \( \tau_y \), and \( \tau_{x'} \), \( \tau_{y'} \). Following the GO approximation (Bass & Fuks, 1972; Valenzuela, 1978), NRCS at incidence angle θ is

\[
\sigma^0(\theta) = |R_{\text{eff}}(\theta)|^2 \frac{w(\tau_x, \tau_y)}{\cos^4 \theta},
\]

where |\( R_{\text{eff}}(\theta) \)| is the effective reflection coefficient, and \( w(\tau_x, \tau_y) \) is the bi-dimensional probability density function (PDF) of sea surface slopes. From estimates for |\( R_{\text{eff}}(\theta) \)| made in Freilich and Vanhoff (2003) it follows that |\( R_{\text{eff}}(\theta) \)| has a slight dependence on wind speed. In particular, for the wind speed in the range 5–15 m/s it varies from 0.45 to 0.5. The angular dependence of |\( R_{\text{eff}}(\theta) \)| is even weaker. Thus we can further replace |\( R_{\text{eff}}(\theta) \)| by |\( R_{\text{eff}}(0) \)|. The slope PDF is quasi-Gaussian and can be approximated by the Gram-Charlier expansion

\[
w(\tau_{x'}, \tau_{y'}) = w_{\text{gauss}}(\tau_{x'}, \tau_{y'}) \left\{ 1 - \frac{1}{2} \varepsilon_{x'}(\varepsilon_{x'} - 1) + \frac{1}{6} \varepsilon_{x'}^3 (\varepsilon_{y'} - 3) + \ldots \right\},
\]

where \( \varepsilon_{x'} = \tau_{x'}/\sigma_{x'x'} \), \( \varepsilon_{y'} = \tau_{y'}/\sigma_{y'y'} \) (\( \sigma_{x'x'} \) and \( \sigma_{y'y'} \) are the mean square slopes along X' and Y').

In the general case, when the incidence plane is oriented arbitrarily with respect to the along and across wind directions, the bi-dimensional normal Gaussian PDF is as follows

\[
w_{\text{gauss}}(\tau_x, \tau_y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-K_{xy}^2}} \exp \left\{ -\frac{1}{2\left(1-K_{xy}^2\right)} \left[ \frac{\tau_x^2}{\sigma_x^2} - 2K_{xy}\frac{\tau_x\tau_y}{\sigma_x\sigma_y} + \frac{\tau_y^2}{\sigma_y^2} \right] \right\},
\]

where \( \sigma_x^2 \) and \( \sigma_y^2 \) are \( \text{mss}_{x,y} \) along X and Y respectively and \( K_{xy} \) is a correlation coefficient between slopes along X and Y. The coefficients \( c_{xy} \) and \( K_{xy} \) are much smaller than 1 and the corresponding terms in 4 and 5 can be neglected. The impact of this assumption on the retrieval accuracy of \( \sigma^0(\theta) \) is concidered in section 5. The final expression of \( \sigma^0(\theta) \) for PR data processing is obtained by putting \( c_{xy} = 0 \) in 4, and \( \tau_x = \tan \theta \), \( \tau_y = 0 \), \( K_{xy} = 0 \) in 5

\[
\sigma^0(\theta) = \frac{|R_{\text{eff}}(0)|^2}{\cos^4 \theta} \exp \left\{ -\frac{\tan^2 \theta}{2\sigma_x^2} \right\}. \quad (6)
\]

From 6 NRCS at nadir can be directly obtained at θ = 0.
$$\sigma^0(\theta) = \frac{|R_{\text{eff}}(0)|^2}{2\sqrt{\sigma_s^2 \sigma_x^2}}.$$  

(7)

hence

$$\sigma^0(\theta) = \sigma^0(0) \exp \left[-\frac{\tan^2\theta}{2\sigma_x^2}\right].$$  

(8)

The analysis from Karaev et al. (2012) was used to restrict the boundaries of applicability of the GO approximation. This is seen to be consistent with experimental data for incidence angles up to $12^\circ$, for a range of wind speeds greater than 3 m/s, and if the contribution of Bragg backscatter is not taken into account. Thus the maximum value of incidence angle for this approximation is $12^\circ$.

4. Method to Extract NRCS at Nadir and $\sigma_x^2$ From PR Data

Now let us turn to the algorithm for retrieving mssLS within the radar swath. Let us call the antenna footprint a cell, and a rectangular area $m$ by $n$ cells a window ($m$ measurements along the flight direction at $n$ angles along the scanning direction). First let us confine the part of the swath used for processing. The PR swath contains 49 cells. Data for incidence angles exceeding $12.5^\circ$ are not taken into account, thus data from the edges of the swath are discarded and only data from the 9th to 41st cells used. In Figure 3 the dependence of NRCS on incidence angle is presented for two successive scans. To eliminate the effect of noise, the PR data were processed within the window, which contains several scans. In the middle of each window the two parameters $\sigma^0(0)$ and $\sigma_x^2$ were defined. In scatterometry, the typical size of the Wind Vector Cell (WVC) varies from 50 km for open ocean to 25 km for coastal regions. Thus the size of the window for PR data processing is 5 cells (about 25 km) across the swath and 5–9 cells (about 25–45 km) along the swath. Within this window wind and waves are considered to be homogeneous. The swath is processed for overlapped windows so that adjacent windows are shifted relative to each other by one cell. As a result, the swaths for $\sigma^0(0)$ and $\sigma_x^2$ were obtained over 29 cells width each, corresponding to 145 km.

Near nadir the NRCS decreases with increase of incidence angle slower than for higher incidence angles (Figure 3). Therefore the influence of noise on the retrieval accuracy and the error in retrieving of $\sigma^0(0)$ and $\sigma_x^2$ are at a maximum in the center of the swath. In order to improve the quality of retrieved $\sigma^0(0)$ within the swath for each scan the retrieved values of $\sigma^0(0)$ in the middle of the swath for the cells 24, 25 and 26 where $\theta < 1^\circ$ were replaced by directly measured (not processed) $\sigma^0(0)$. The method for $\sigma^0(0)$ and $\sigma_x^2$ retrieval is based on equation (8). Usually, as in Freilich and Vanhoff (2003), Chu and Chen (2010), Nouguier et al. (2016), the parameters are obtained by solving the least squares problem. Below we discuss the application of the least squares regression method, and an alternative method is suggested.

4.1. Method A

By taking the logarithm, the equation (8) is converted to linear form:

$$\ln (\sigma^0(\theta) \cos^4 \theta) = \ln (\sigma^0(0)) - \tan^2 \theta / 2\sigma_x^2.$$  

(9)

After linear regression, the parameters $\sigma^0(0)$ and $\sigma_x^2$ can be obtained. One window contains NRCS at $n$ angles, and for each angle there are $m$ samples of NRCS. The example of dependence of $\ln (\sigma^0(\theta) \cos^4 \theta)$ (by $\sigma^0$ "noisy" NRCS for each cell is denoted) on $\tan^2 \theta$ for one window is presented in Figure 4.

The quality control procedure was conducted before linear regression. First, if for any cell in the window the rain flag differed from zero the whole window was discarded. Second, the outliers were removed from the data set for each window. Thereafter the quantity of the remaining data was checked. Thresholds for the minimum quantity of different incidence angles and measurements for each angle were established, with at least 2 measurements for 2 incidence angles required. For further processing, the minimum quantity of

![Figure 3. The dependence of NRCS on incidence angle for two successive scans.](image-url)
angles was set equal to 4 and minimum quantity of measurements for each angle was also set equal to 4. If the quantity of the data in the window is insufficient, the window is discarded.

Finally, the correlation coefficient between \( \ln(\sigma^0(\theta)\cos^4\theta) \) and \( \tan^2(\theta) \) was calculated. If the absolute value of correlation coefficient is low, the waves are not homogeneous within the window, or the level of noise is rather high. The correlation coefficient is calculated as follows

\[
K = \frac{\left(\left(\ln(\sigma^0(\theta)\cos^4\theta) - \ln(\sigma^0(\theta_0)\cos^4\theta)\right)\left(\tan^2\theta - \tan^2\theta_0\right)\right)}{\sqrt{\left(\left(\ln(\sigma^0(\theta)\cos^4\theta) - \ln(\sigma^0(\theta_0)\cos^4\theta)\right)^2\left(\tan^2\theta - \tan^2\theta_0\right)^2\right)}},
\]

where the top dash denotes averaging within the window. Due to the angular dependence of NRCS, \( K \) should be negative. If it is positive, or if it’s absolute value is lower than the threshold (0.7 obtained in the numerical experiment), the window is discarded. The aim of this procedure is to exclude from consideration highly inhomogeneous regions, where the parameters cannot be precisely determined.

4.2. Method B

It is enough to have measurements at two different incidence angles, \( \theta_1 \) and \( \theta_2 \), to obtain \( \sigma^0(\theta) \) and \( \sigma^2(\theta) \). From 8 it follows that

\[
\begin{align*}
\ln(\sigma^0(\theta_1)\cos^4\theta_1) &= \ln(\sigma^0(\theta_0)\cos^4\theta_0) - b\tan^2\theta_1, \\
\ln(\sigma^0(\theta_2)\cos^4\theta_2) &= \ln(\sigma^0(\theta_0)\cos^4\theta_0) - b\tan^2\theta_2,
\end{align*}
\]

\[b = \frac{1}{2\sigma^2} \]  

For an \( m \) by \( n \) window (\( m \) measurements at \( n \) angles), the average NRCS at each incidence angle was calculated as the first step. The array \( \left( \theta, \sigma^2 \right) \) of \( n(n-1)/2 \) pairs was then formed, and \( b \) calculated for each pair. The example of selection of pairs is presented in Figure 5.

For the next step the outliers removal procedure was carried out. If the quantity of remaining elements in the array \( b \) was lower than the threshold value (4 for calculations in this paper), the window was discarded. Otherwise, the \( \sigma^2 \) was calculated in two ways: first by

\[\sigma^2_{(1)} = \frac{1}{2\text{median}(b)}\]

and second by

\[\sigma^2_{(2)} = \text{median}\left(\frac{1}{2b}\right)\]

where median denotes median value of the array. Another option would be to average \( b \) and \( 1/b \), but it was found that using median values achieved better accuracy. The arrays of \( \sigma^2_{(1)}(0) \) and \( \sigma^2_{(2)}(0) \) for each incidence angle were calculated by substituting \( \sigma^2_{(1)}(0) \) and \( \sigma^2_{(2)}(0) \) into 7. The two estimates of \( \sigma^2_{(1)}(0) \) and \( \sigma^2_{(2)}(0) \) were obtained as follows

\[
\begin{align*}
\sigma^2_{(1)}(0) &= \text{median}\left(\hat{\sigma}^2_{(1)}(0)\right), \\
\sigma^2_{(2)}(0) &= \text{median}\left(\hat{\sigma}^2_{(2)}(0)\right).
\end{align*}
\]

The resulting values of \( \sigma^0(0) \) and \( \sigma^2 \) were calculated by averaging the two estimates.
The quality control of the data are introduced by estimating the effect of the correlation coefficient $r$. For wind speed from 2 to 16 m/s, the calculation was performed for wind speed equal to 16 m/s when the coefficients are obtained experimentally in Chu et al. (2012b). These parameters were obtained for a wind field retrieval because it allows for abrupt changes in wind field, which can be associated with atmospheric fronts (Shaffer et al., 1991). The swaths of retrieved $\sigma^2(0)$ and $\sigma_0^2$ contain gaps for those windows where the data quality is insufficient. During the second stage of processing, a median filtering was performed in order to eliminate noisy data and to fill the gaps. Median filtering is used in scatterometry for wind field retrieval, because it allows for abrupt changes in wind field, which can be associated with atmospheric fronts (Shaffer et al., 1991). The swaths of retrieved $\sigma^2(0)$ and $\sigma_0^2$ were filtered by the window of 5 x 5 cells. In the midpoint of the window the value of $\sigma^2(0)$ or $\sigma_0^2$ was replaced (if the value was defined there) or filled (if there was a gap) by the median value of all the defined values within the window if their quantity exceeded a half of the total amount of cells in the window (for window 5 x 5 cells the threshold is 13). If the defined cells occupied less than a half of the window, the value or the gap remained.

4.3. Median Filtering

After the first stage of processing, by either A or B method, the swaths of retrieved $\sigma^2(0)$ and $\sigma_0^2$ contain gaps for those windows where the data quality is insufficient. During the second stage of processing, a median filtering was performed in order to eliminate noisy data and to fill the gaps. Median filtering is used in scatterometry for wind field retrieval, because it allows for abrupt changes in wind field, which can be associated with atmospheric fronts (Shaffer et al., 1991). The swaths of retrieved $\sigma^2(0)$ and $\sigma_0^2$ were filtered by the window of 5 x 5 cells. In the midpoint of the window the value of $\sigma^2(0)$ or $\sigma_0^2$ was replaced (if the value was defined there) or filled (if there was a gap) by the median value of all the defined values within the window if their quantity exceeded a half of the total amount of cells in the window (for window 5 x 5 cells the threshold is 13). If the defined cells occupied less than a half of the window, the value or the gap remained.

5. Numerical Simulation

Currently, ocean wave buoys do not measure $\text{mss}_{22}$. Thus the only way to validate the algorithm for retrieval of $\sigma^2(0)$ and $\sigma_0^2$ is through numerical simulation, as described in this section. First, the assumption for dependence of $\sigma^2(0)$ was checked. The errors connected with neglect of $c_{pq}$ and $K_{xy}$ were evaluated separately. To estimate the effect of coefficients $c_{pq}$, the maximum values of $c_{03}$, $c_{21}$, and $\sigma_0^2$ were taken from Chu et al. (2012b). These parameters were obtained experimentally in Chu et al. (2012b) from PR data for a wind speed from 2 to 16 m/s. The calculation was performed for wind speed equal to 16 m/s when the coefficients $c_{03}$ and $c_{21}$ take maximum values. The dependence of $\sigma^2(0)$ was calculated for $c_{03}=0.1$, $c_{21}=0.04$, $\sigma_0^2=0.02$, and $\sigma_0^2=0.02$. The parameters $\sigma^2(0)$ and $\sigma_0^2$ were calculated according to equation 4.2 as with $c_{03}$ and $c_{21}$ set to zero. Let us define the relative error of the parameter $\sigma$ as $\delta\sigma=|\sigma-\hat{\sigma}|/\sigma_0$, where $\sigma_0$ and $\sigma$ are the true and the retrieved value respectively. In this case, the relative error is less than 0.05 for $\sigma_0^2$ and is less than 0.01 for $\sigma^2(0)$. Therefore $c_{pq}$ can be neglected, and the PDF of sea waves slopes can be considered to be Gaussian in further calculations.

To estimate the effect of correlation coefficient $K_{xy}$, $\sigma^2(0)$ was calculated for azimuthal angle between the scanning direction and the direction of wave propagation from 0 to 180°. The parameters $\sigma^2$, $\sigma_0^2$, and $K_{xy}$ were calculated for different wind speeds from the model wind wave spectrum (Karaev et al., 2008) with boundary wave number $k_0$ for Ku-band obtained in Panfilova and Karaev (2016). The parameters $\sigma^2(0)$ and $\sigma_0^2$ were retrieved from equation (6) setting $K_{xy}$ equal to zero. The relative error was averaged over all azimuthal angles. At wind speed equal to 3 m/s it is equal to 0.017 for $\sigma^2(0)$ and to 0.035 for $\sigma_0^2$. For higher wind speeds the error is less.
Also, the effect of noise in the PR data on the quality of parameter retrieval was investigated. To study the behavior of NRCS in real data, the homogeneous part of the 500 km long PR swath was selected. For each incidence angle, the mean value of NRCS, \( \bar{\sigma}^0 (\theta) \), and the array of deviations from the mean \( \bar{\sigma}^0 (\theta) - \sigma^0_{\text{NRCS}} (\theta) \) were calculated. The distribution of noise for a given angle is close to Gaussian with variance \( \sigma_{\text{noise}} \) equal to the variance of \( Inoise \) for the selected realization. The dependence of \( \sigma_{\text{noise}} \) on incidence angle is shown in Figure 6 for \( \sigma^0_{\text{NRCS}} (0) = 12 \) dB. This value was chosen as representing typical value for NRCS at nadir.

The distribution of \( \sigma_{\text{noise}} \) from Figure 6 was used in the numerical experiment. The NRCS was calculated at specified values of \( \bar{\sigma}^0 (0) \) and \( \sigma^2_x \) with additive random noise \( Inoise \) with Gaussian distribution and variance \( \sigma_{\text{noise}} \) as in Figure 6.

\[
\bar{\sigma}^0 = \bar{\sigma}^0 (\theta, \bar{\sigma}^0 (0), \sigma^2_x) + Inoise.
\]

(20)

The discretization of NRCS values mentioned in section 2 should be taken into account in the numerical simulation. In order to simulate this discretization, the following transformation is carried out:

\[
\bar{\sigma}^0_{\text{db}} = 0.35 \left( \frac{\bar{\sigma}^0_{\text{db}}}{0.35} \right) + 0.175.
\]

(21)

where \([ \ldots ]\) denotes the integer part.

An example of the simulated swath \( \bar{\sigma}^0_{\text{db}} (\theta) \) is given in Figure 7a. The distribution of retrieved \( \sigma^0_{\text{db}} (0) \) before and after median filtering is shown in Figures 7b and 7c. White regions here show the gaps where data were discarded. In Figure 7b three lines in the middle of the swath for the cells 24, 25 and 26 are replaced by directly "measured" values of \( \bar{\sigma}^0_{\text{db}} (\theta) \).

Simulation of \( \bar{\sigma}^0_{\text{db}} (\theta) \) was performed for a part of radar swath. The number of scans is 1,000 and incidence angle varied from 0 to 12° (17 cells for each scan). The swath was processed by overlapped windows 5 × 5 cells. The true value of \( \sigma^2_x \) varied from 0.005 to 0.025 and the parameters \( \bar{\sigma}^0 (0) \) and \( \sigma^2_x \) were retrieved. The error was estimated before and after median filtering. The error was found to grow with increasing \( \sigma^2_x \) and its maximum estimates are presented and discussed below. The relative bias for \( \bar{\sigma}^0 (0) \) (in natural units) before and after median filtering is less than 0.01 and less than 0.006 respectively, and the relative root-mean-square error (rmse) is less than 0.11 and less than 0.08 respectively; for \( \sigma^2_x \) the relative bias before and after median filtering is less than 0.11 and less than 0.08 respectively.
after median filtering is less than 0.05 and less than 0.005 respectively, and the relative rmse is less than 0.4 and 0.2 respectively. These estimates show that median filtering improves quality of the retrieved parameters. Thus the numerical simulation confirmed the validity and accuracy of the developed algorithm.

6. Total Mean Square Slope

Let us consider the parameters $\sigma^0(0)$ and $\sigma_2^0$. The mean square slope $\sigma_2^0$ is measured along the scanning direction, which is not connected to the wind or wave propagation direction. Variations of $\sigma_2^0$ can be caused by both variations of wave direction and intensity of waves within the radar swath. The mean square slope along scanning direction $\sigma_2^0$ can be within the range from $\sigma_2^0$ to $\sigma_2^0$ (where $\sigma_2^0$ is the mean square slope along wind, and $\sigma_2^0$ is the mean square slope across wind direction respectively). The value of $\sigma^0(0)$ does not depend on the wave propagation direction, and therefore it is a more universal parameter which should depend on the total $msss_{LS}$.

The value of the total mean square slope $\sigma_{tot}^2=\sigma_{x}^2+\sigma_{y}^2=\sigma_{x}^2+\sigma_{y}^2$ lies between $2\sigma_2^0$ and $2\sigma_2^0$ as well as $2\sigma_2^0$ retrieved from PR data. Therefore in statistical meaning $\sigma_{tot}^2$ behaves as $2\sigma_2^0$.

The data for the Atlantic and Pacific oceans and the Gulf of Mexico were processed to provide coverage of a wide variety of wave conditions. For all the data set, 4,075,32 pairs ($\sigma_2^0$, $\sigma^0(0)$) were retrieved and the dependence $2\sigma_2^0(\sigma^0(0))$ was obtained. Using non-linear regression the dependence of $\sigma_{tot}^2(\sigma^0(0))$ was obtained, as follows:

$$\sigma_{tot}^2=0.4465/\sigma^0\pm0.0049,$$ (22)

The equation is valid for $\sigma^0(0)$ within the range from 8 to 250 (natural units). Experimental data for $2\sigma_2^0$ ($\sigma^0(0)$) is presented in Figure 8 as well as regression curve $\sigma_{tot}^2(\sigma^0(0))$.

To summarize, the distribution of $\sigma_{tot}^2$ can be obtained in two steps: 1) the distribution of $\sigma^0(0)$ in the swath is obtained by method A or B, and median filtering procedure is carried out, then 2) the distribution of $\sigma_{tot}^2$ is calculated in the swath by the equation (22).

7. Distribution of NRCS Within PR Swath: Contrast on a Surface Covered by an Oil Film

For the qualitative check of the PR data processing algorithm, data over a large oil spill was used. It is known that an oil film on the surface suppresses short waves, which leads to decrease of $msss_{LS}$ and increase of $|R_{ref}|^2$.

According to the equation (5) NRCS trend would change due to variations in $\sigma_2^0$. An increase in $\sigma_2^0$ leads to a decrease of NRCS at nadir and an increase in NRCS at the edge of the swath and vice versa: a decrease in $\sigma_2^0$ leads to an increase of NRCS at nadir and a decrease at the edge of the swath for incidence angles lower than approximately 12°. For incidence angles higher than 12° resonant scattering mechanism and breaking waves should be taken into account. An oil film suppresses waves of cm scale, and thus the spectrum density of resonant ripples and NRCS decrease.

In previous papers on PR and Dual-frequency Precipitation Radar (DPR) data the dependency of NRCS on the incidence angle is presented at different wind speeds (Chu et al., 2012a; Freilich & Vanhoff, 2003) and at various significant wave heights (SWH; Nouguier et al., 2016). These studies show that the dependence $\sigma^0(\theta)$ is stronger at lower sea states than at higher sea states. This variation in NRCS trend is due to the correlation of $\sigma_2^0$ with SWH and wind speed. Also in case of oil spill the dependence $\sigma^0(\theta)$ is stronger than in case of clean surface.

It was shown in Chu et al. (2012a) that at an incidence angle of about 5–11° (depending on the sea state) the correlation of NRCS with wind speed and surface roughness parameters is low. Thus changes of the
signal in the middle of the swath (incidence angles less than 5°) and on the edge of the swath (incidence angles greater than 10°) indicate variations of surface roughness most clearly.

In Figure 9 the spatial distribution of NRCS within the PR swath is presented accompanied by a MODIS image of the oil film on the surface of the Gulf of Mexico. The long stripe of oil slick is crossed by the PR swath. In Figure 9b the slick is in the zone of Sun’s glitter and is lighter than the background. In Figure 9a the red color in the middle of the swath and the blue color on the edge indicate the area smoothed by the oil slick.

Figure 9. The distribution of NRCS within the PR swath obtained on the 17 May 2010 at 12:38 UTC, wind speed 3.1 m/s (a). The fragment of MODIS Terra image. The image was obtained the same day at 16:40 UTC. The lines a, b, c, and d denote cuts along the swath presented in Figure 10.

Figure 10. The dependence of NRCS on the distance along the track for incidence angles (a) 1.5°, (b) 16.6°, (c) −16.6°, and (d) 9°.
At $\theta = 1.5^\circ$ NRCS increased by approximately 8 dB (Figure 10a), at $\theta = 16.6^\circ$ decreased by approximately 30 dB (Figure 10b), and at $\theta = 9^\circ$ remains the same (Figure 10c).

In Figure 10b the two drops in NRCS are observed. In the optical image only one slick in the zone of Sun’s glitter is clearly visible (Figure 9b), however another one is visible in PR image (Figure 9a).

The value of contrast, calculated as the ratio of NRCS from oil-covered surface and from the clean surface, is used to explore the effect of the oil film on the backscatter. For the period from April 2010 till July 2010 several parts of the PR swath were selected, and the dependence of contrast on the incidence angle obtained for wind speeds of 3 m/s and 5.4 m/s (see Figure 11). The value of $\sigma_0^0$ is the average value of $\sigma_0^0$ (in natural units) for a “clean” surface and $\sigma_0^{o}$ is the average value for oil-covered surface. The standard deviations of $\sigma_0^0$ from $\sigma_0^0$ and $\sigma_0^{o}$ were used as the error of contrast.

The values of the contrast seen in Figure 10 (e.g. 35 dB in Figures 10b and 10c) are higher than shown in Figure 11. The maximum contrast occurs in the middle part of the slick where the oil thickness is higher than at the edges. Therefore, the contrast for averaged NRCS is lower than for peak values.

The behavior of contrast is in a good agreement with Johnson and Crosswell (1982) which presents dependencies for the same band and polarization. In Garnakeryan and Sosunov (1978) the experimental results for X-band and vertical polarization are presented. The absolute value of contrast at X-band is seen to be higher than for Ku-band and horizontal polarization.

8. Mean Square Slope in the Slick

In this section, the method for total $mss_{\text{LS}}$ retrieval is applied to the PR data over the Gulf of Mexico for the period of the oil spill. Figure 12 b) presents the initial radar image (NRCS) measured by PR (before processing) for 9 June 2010. Figure 12a presents the MERIS Envisat image for comparison. In Figures 12c and 12d the distributions of $\sigma_0^0$ and $\sigma_0^{2_{\text{tot}}}$ are shown.

Interpretation of the PR image before processing is difficult, because the NRCS is affected by both angular dependence and changes in surface roughness. However, the distribution of $\sigma_0^2(0)$ and $\sigma_0^{2_{\text{tot}}}$ contains information on surface roughness only. In the Figures 12c and 12d the shape of the oil film can be seen more clearly. In the region of oil spill, which was detected by the optical image, the value of $\sigma_0^2(0)$ increases by 3 dB and the value of $\sigma_0^{2_{\text{tot}}}$ decreases four times compared to the background.

The data on $\sigma_0^{2_{\text{tot}}}$ were combined with the buoy data on wind speed. The buoy number 42040 is located 54 km from the platform. The anemometer is installed at 4 m height, and data on wind speed and direction are provided each 10 minutes. For further analysis the wind speed at 10 m height was calculated by using the formulae for neutral stratification (Masuko et al., 1986).
The time window between buoy and PR measurements is within 10 minutes, and the estimate of $r^{2}_{\text{tot}}$ was calculated as an average within 15 km around the buoy. Though wind speed and waves are homogeneous at greater distances, a 15 km window was chosen to remain within the area of the slick. Consequently, the quantity of data collocated with buoy is not large. The data set contains 17 qualitative observations for the period from the end of April to the beginning of July 2010. Optical images were used to select samples where the presence of oil spill near the buoy was confirmed. Also, data for May–November 2012 in the same region for a “clean” surface (i.e. without a slick present) were processed using the same algorithm.

The experimental data and regression dependencies of $r^{2}_{\text{tot}}$ on $U_{10}$ for oil-covered and clean surface are presented in Figure 13a. The derived dependence for a clean surface,

$$r^{2}_{\text{totKu, clean}} = 0.0111 + 0.002U_{10} \pm 0.006$$  \hspace{1cm} (23)

is in a good agreement with Chu et al. (2012b) and Nouguier et al. (2016). The dependence for slick in Ku-band is obtained for the first time here as

$$r^{2}_{\text{totKu, slick}} = 0.0071 + 0.0018U_{10} \pm 0.004.$$  \hspace{1cm} (24)

To our knowledge, these represent the only measurements of total $mss_{LS}$ for Ku-band in a slick available to date. Therefore, we compared the result with optical experiment by Cox and Munk (1954). Experimental data and the regression dependence for the optical case are shown in Figure 13b.

For clean water the difference between total $mss_{LS}$ (Ku-band) and the mean square slope from the Cox and Munk experiment is large (dashed lines in Figure 13), whereas for oil-covered surface the values agree quite well (solid lines in Figure 13).

The mean square slope for clean surface from the optical experiment is given by:

\[ \sigma^2_{\text{totKu, clean}} = 0.011 + 0.002U_{10} \pm 0.006 \]
\[ r_{\text{tot}}^{2} = \int_{0}^{\infty} k^2 S(k) \, dk, \]  
(25)

and in Ku-band the radar-filtered mssLS is given by:

\[ r_{\text{tot}}^{2} = \int_{0}^{\infty} k^2 S(k) \, dk, \]  
(26)

where \( S(k) \) is the spectrum for clean surface. An oil film on the surface suppresses short waves for which \( k > k_s \), thus the spectrum of sea wave in the area of slick can be approximated by

\[ S_{\text{slick}}(k) = S(k) \quad \text{if} \quad k \leq k_s, \]
\[ S_{\text{slick}}(k) = 0 \quad \text{if} \quad k > k_s. \]  
(27)

The mean square slope for oil-covered surface from the optical experiment is as follows

\[ r_{\text{tot optics}}^{2} = \int_{0}^{\infty} k^2 S(k) \, dk, \]  
(28)

and in Ku-band the radar-filtered mssLS is

\[ r_{\text{tot Ku slick}}^{2} = \int_{0}^{\min(k_b, k_s)} k^2 S(k) \, dk. \]  
(29)

According to the Figure 13a), \( r_{\text{tot Ku clean}}^{2} > r_{\text{tot Ku slick}}^{2} \), which means that \( k_b > k_s \), and

\[ r_{\text{tot Ku slick}}^{2} = \int_{0}^{k_s} k^2 S(k) \, dk. \]  
(30)

and that is why for a slick covered surface, the mean square slope for optics and for Ku-band is almost the same.

9. Conclusion

The algorithm for calculation of NRCS at nadir and \( r_{\text{tot}}^{2} \) from PR Ku-band swath data has been described. The distribution of \( r_{\text{tot}}^{2} \) allows the observation of large oceanic and atmospheric processes which are accompanied by changes of surface roughness, such as typhoons, currents, atmospheric fronts. In this paper this algorithm have been applied for investigations of oil spills. We have seen that \( r_{\text{tot}}^{2} \) is an additional parameter of surface waves, besides significant wave height, which can be measured from space. There is a problem however in the interpretation of \( r_{\text{tot}}^{2} \), because the behavior of mssLS also depends on the radar wavelength. Despite this, relative variations of \( r_{\text{tot}}^{2} \) for measurements at one frequency contain new
information on surface waves. Also the dependence of $\sigma^2_{tot}$ on wind speed in the presence of a surface slick has been obtained. Comparison with Cox and Munk data revealed that, within the slick, Ku-band slopes coincide with optical mean square slope, and can hence be used for investigation of suppression of short waves by the film.

Acknowledgments
This work is supported by international cooperation, CAS, chinese-foreign cooperation in key projects "The detection of oil spill and its ecological impact study" (133337KYSB20160002), RFBR grants 16-35-00548 mol.a, 15-45-02-50 r_povolzhye_a, and the National Natural Science Foundation of China (41576032). We would like to thank David Cotton (Satellite Oceanographic Consultants Limited) who helped to significantly improve the manuscript and anonymous reviewers for valuable comments and discussion. The data necessary to reproduce the results are available from https://figshare.com/articles/panfilova_rar/5686639.

References